Unit: Measurement: 2-D & 3-D

Grade 9 Applied Mathematics

Lesson Outline

Overall Expectation
Students will:
• Solve problems involving the measurements of two-dimensional shapes and the volumes of three-dimensional figures;
• simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations;
• learn about global peace as it relates to safe drinking water through problem solving.

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<th>Math Learning Goals</th>
<th>Expectations</th>
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<td>1</td>
<td>Fill it Up</td>
<td>• Develop through investigation the formulas for volume of a pyramid, and a cone based on the volume of the corresponding prism or cylinder of the same radius and height.</td>
<td>MG2.04, NA2.01, NA2.03, NA2.04, NA2.08</td>
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<td>2</td>
<td>A Global Problem</td>
<td>• Develop through investigation the formula for volume of a sphere based on the volume of a cylinder/cone.</td>
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<td>• Consolidate volumes of prisms, pyramids, cylinders, cones and spheres.</td>
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<td>• Solve problems involving combinations of the figures using metric and imperial measure.</td>
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Teaching/Learning Strategies
Discussion
Homework
Think/Pair/Share
Internet Technologies
Experimenting
Manipulatives
Media Presentation

Student Groupings
Students working as a whole class
Students working in pairs
Students working in small groups
Students working individually

Assessment Strategies
Observation

Assessment Recording Devices
Anecdotal Record
Checklist

Adaptations
Special need: Hearing impaired, which is decreased ability to perceive auditory information. Have the student sit at the front of the class. During group discussions, only one person can talk at a time. Both students and teacher must speak clearly and audibly for everyone, in any location of the classroom, to hear. When watching the video during day one, have a headset available for this student. After group discussions and lectures, verify that the student is clear on what was being discussed.

Prior Knowledge
Students must have completed grade 8 Mathematics. They must know how to find the perimeter, surface area, and area of figures such as squares, circles, and triangles.
Day 1: Fill it up

**Math Learning Goals**
- Develop through investigation the formulas for volume of a pyramid, and a cone based on the volume of the corresponding prism or cylinder of the same radius and height.

**Whole class __ Discussion**
Introduce Global Peace by asking what it means to the students. Ask how safe drinking water is an aspect of Global Peace. (10mins)

**Whole class __ Watch Video & Discuss**
Play online video about providing clean drinking water to third world countries. (10mins)
(www.theh2oproject.org/video_windowsmedia.html)

**Small Groups __ Exploration**
Each group works with one set of solids. Orient students to the 3-D relational solids. Allow for exploration time. Students establish their own criteria and rationale for sorting the various solids into groups. (5mins)

**Whole Class __ Sharing**
Each group shares its strategy. Use this opportunity to review the terminology related to the geometric shapes and how each shape can be either a container or glass to hold water. (10mins)

**Small Groups __ Investigation**
The groups compare the volumes of prisms (cylinder) and pyramids (cone) with congruent bases and equal heights by pouring water in the pyramids and discovering how many times it takes to fill the corresponding prisms. (see 1.1.1). (15mins)

**Whole Class __ Journal: Representing**
Give the class the following journal prompt: Using words, pictures, numbers, and symbols, describe the relationships you discovered today. (10mins)

**Pairs __ Pair/Share**
Students complete 1.1.2, alternating as indicated between writing and coaching. (15mins)

**Explore Reflection**
**Home Activity**
- Discover/Record 3 other organizations that help provide safe drinking water to those in need.
- Find a drinking glass and determine its volume with any method. Be prepared to share your discoveries with class.

**Materials**
- 3-D relational solids
- video
- work sheets (1.1.1 & 1.1.2)
- water
- napkins
- computer projector

Assess through observation (with a rating scale) how students interact as they investigate in their groups.
Day 2: A Global Problem

Math Learning Goals
• Develop through investigation the formula for volume of a sphere based on the volume of a cylinder/cone.
• Consolidate volumes of prisms, pyramids, cylinders, cones and spheres.
• Solve problems involving combinations of the figures using metric and imperial measure.

Whole class _ Discussion
Discuss home activity given from last class. Have students share the different organizations that they discovered. (10mins)

Pairs _ Think/Pair/Share/Discussion
Individually, students make hypotheses about the relative volumes of pairs of 3-D solids. In pairs, they compare and consolidate their lists. Ask:
• Are there any other pairs where one is 3 times the other? 4 times? 2 times? [hemisphere _ 3 = cylinder; small, triangular-based prisms _ 4 = large, triangular prism; small cylinders _ 2 = large cylinder;]
• Is there a series of shapes which all compare? [square and rectangular-based prisms (all 4 doubling each time)]
• Are there any shapes with equal volume? [hemisphere = cone]
Demonstrate selected examples, emphasizing the connection between the hemisphere and cone. (10mins)

Pairs _ Investigation
Students complete 1.2.1 using the 3-D relational solids. They read the problem and highlight important information needed to solve the problem. (10mins)

Whole Class _ Presentation
To develop further understanding of the formula of the volume of a sphere, show the electronic presentation, Developing the Formula for the Volume of a Sphere, which formalizes the relationship between the volume of a sphere and a cone. Volume-Sphere.ppt (25mins)

Pairs _ Pair/Share
Students solve a variety of problems involving the volume of prisms, pyramids, and spheres (see 1.2.2). (20mins)

Home Activity
• Discover the volume of the world online and therefore determine what the radius of the world is through manipulation.
• Chose one of the organizations that provide safe drinking water and discover what containers are used to distribute water to families in need. Choose one container, estimate the dimensions, and present a complete solution for the volume of the structure.

Assess through observation (with an anecdotal record) how pairs discuss to determine their understanding of the relationships being investigated.

Materials
• 3-D relational solids
• computer projector
• work sheets (1.2.1 & 1.2.2)

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Foundations of Mathematics, Grade 9, Applied Curriculum

Measurement and Geometry (MG)
Solving Problems Involving Perimeter, Area, and Volume (MG2)

MG2.01 – relate the geometric representation of the Pythagorean Theorem and the algebraic representation $a^2 + b^2 = c^2$;

MG2.02 – solve problems using the Pythagorean Theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);

MG2.03 – solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles) (Sample problem: A new park is in the shape of an isosceles trapezoid with a square attached to the shortest side. The side lengths of the trapezoidal section are 200 m, 500 m, 500 m, and 800 m, and the side length of the square section is 200 m. If the park is to be fully fenced and sodded, how much fencing and sod are required?);

MG2.04 – develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is $1/3$ the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = V_{\text{prism}}/3$ or $V_{\text{pyramid}} = ((\text{area of base})(\text{height}))/3$);

MG2.05 – solve problems involving the volumes of prisms, pyramids, cylinders, cones, and spheres (Sample problem: Break-bit Cereal is sold in a single-serving size, in a box in the shape of a rectangular prism of dimensions 5 cm by 4 cm by 10 cm. The manufacturer also sells the cereal in a larger size, in a box with dimensions double those of the smaller box. Make a hypothesis about the effect on the volume of doubling the dimensions. Test your hypothesis using the volumes of the two boxes, and discuss the result.).

Mathematics Number Sense and Algebra (NA)
Simplifying Expressions and Solving Equations (NA2)

NA2.01 – simplify numerical expressions involving integers and rational numbers, with and without the use of technology;* *The knowledge and skills described in this expectation are to be introduced as needed and applied and consolidated throughout the course.

NA2.02 – relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations;

NA2.03 – describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three (i.e., length, which is one dimensional, can be represented by $x$; area, which is two dimensional, can be represented by $(x)(x)$ or $x^2$; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or $x^3$);

NA2.04 – substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases) [e.g., evaluate $(3/2)^3$ by hand and 9.83 by using a calculator] (Sample problem: A movie theatre wants to compare the volumes of popcorn in two containers, a cube with edge length 8.1 cm and a cylinder with radius 4.5 cm and height 8.0 cm. Which container holds more popcorn?)* *The knowledge and skills described in this expectation are to be introduced as needed and applied and consolidated throughout the course.

NA2.08 – substitute into algebraic equations and solve for one variable in the first degree (e.g., in relationships, in measurement) (Sample problem: The perimeter of a rectangle can be represented as $P = 2l + 2w$. If the perimeter of a rectangle is 59 cm and the width is 12 cm, determine the length.).
Student Groupings

Working as a whole class:
Students’ working as a whole class provides opportunity for:
• clarification under teacher direction of a problem, question, issue;
• sharing of ideas and information by many;
• delivery of new required information by an expert;
• creating a common body of knowledge shared by the group.
When using this grouping formation, keep in mind:
• it is not always evident who is really listening and learning;
• it is important to ensure all students participate, not just those who raise their hands or are the first to do so;
• great risk is attached to speaking in front of a whole class, particularly when the students may not know if he or she is right or not.

Working in pairs:
Students’ working as a whole class provides opportunity for:
• clarification under teacher direction of a problem, question, issue;
• sharing of ideas and information by many;
• delivery of new required information by an expert;
• creating a common body of knowledge shared by the group.
When using this grouping formation, keep in mind:
• it is not always evident who is really listening and learning;
• it is important to ensure all students participate, not just those who raise their hands or are the first to do so;
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Working in small groups:
Small group learning is a powerful teaching and learning approach. To be effective in using this strategy, teachers must believe in the importance of language rich environments for learning. The use of small groups for learning involves deciding when to use small groups in the learning process and which small group structure or strategy will be best suited to the learning task. The composition of small groups is also a vital factor in their success.
Students will require assistance from their teachers in learning how to work productively in small groups. For example, students need to learn how to focus discussions, share resources, clarify and delegate tasks, negotiate disagreements, etc. Therefore, the deliberate and thoughtful teaching of collaborative skills is a significant contributor to the success of small group activities, and consequent student achievement. This can be achieved successfully through debriefing sessions led by the teacher.
Small group learning experiences are enhanced when teachers and students reflect on the small group process to form and apply generalizations about what contributes to their success. Students need to be part of the planning, critiquing and analyzing of the mechanics of grouping. Teachers and students can ease gently into small group learning to allow themselves time to build trust and confidence in working together. Starting with pairs working together is frequently a comfortable means of introducing small group work into the classroom for both the teacher and the students. Teachers can incorporate small group activities for short periods within periods of direct instruction or during individual learning activities. These activities might be used:
• to introduce a lesson (for example, partners discussing their prior knowledge);
• to ‘pause for thought’ during a lesson (for example, brainstorming questions in pairs);
• to summarize what has been learned (for example, reviewing three key points with a partner).
These brief opportunities for small group interaction within a period of instruction will be more successful if the focus for discussion is:
• specific
• concrete
• immediately relevant/purposeful
• intrinsically rewarding or followed by prompt feedback.

Working Individually:
Students’ working individually provides opportunity for:
• consolidation of knowledge and skills learned;
• work at one’s own rate;
• individual goal-setting and time management;
• individual conferencing with teacher about questions and progress.
When using this grouping formation, keep in mind:
• close monitoring is needed as some students can appear to be understanding and working productively, when, in fact, they are not.
Teaching/Learning Strategies

Discussion
Discussion is purposeful talk through which students explore thinking, respond to ideas, process information, and articulate their thoughts in verbal exchanges with peers and teachers. Discussion is used to promote and clarify understanding of concepts, ideas, and information in all subject areas. It places the emphasis on students talking and listening to each other. Students use discussion to make connections between ideas and experience and to reflect on a variety of meanings and interpretations of texts, experiences, and phenomena.

Homework
Homework is work that is completed by the student outside of class. Homework can involve review and practice of classroom learning or may be remedial in nature. It may be assigned to encourage students’ responsibility for their own learning and their acquisition of specific skills or knowledge. Homework can also encourage students to pursue an in-depth interest in a topic and generate new thoughts on the topic. Homework may be assigned to individual students or to the class as a whole. It can be informal (for instance, personal reading) or formal (for instance, a specific task or set of questions) in nature.

Think/Pair/Share
Think/pair/share is a strategy whereby students think alone for a specified amount of time (wait time) in response to a question posed by the teacher. Students form pairs to discuss their ideas, and then share responses with the class. Think/pair/share is used to help students check their understanding during a learning experience and provide opportunities for practice or rehearsal. It provides a simple structure within a short time frame for all students in the class to think and talk (to pose questions, to respond to an issue, to summarize or synthesize ideas).

Internet Technologies
The Internet (Net) is a worldwide computer network connecting users to each other for communication. This “network of networks” was originally conceived for academic and military research and now connects educational institutions, private and public services, commercial enterprises, and individuals. Computer users connected to the Internet can read and post messages, download software and media files, research information by browsing directories and following hyperlinks on diverse websites, communicate with experts, and search catalogues of major libraries around the world. The Internet provides an open, unmonitored forum to which anyone can contribute and publish different viewpoints. Because of its open structure, the Internet requires that students learn critical searching and communication skills in order to find and use relevant, valid information in a timely, safe, and ethical manner.

Experimenting
Experimenting involves carrying out investigations to test predictions, hypotheses, or prior knowledge and understanding. It focuses on problem solving and decision making and is a link to real-world, authentic learning. Experiments are central to science, technology, and mathematics, but may also be applied to other disciplines, such as the arts, to make connections between concrete and abstract learning. Experimenting requires that students follow an established procedure, such as the scientific method, which is structured so that students hypothesize, investigate, test, explore, manipulate, and organize information and record their observations. Experimenting also encourages students to use cooperative skills effectively in communication when interpreting experimental findings. Experiments enhance student motivation, understanding, and active involvement and can be initiated by the teacher or the student.

Manipulatives
Manipulatives are materials that appeal to the senses and can be touched, moved about, and rearranged. Working with manipulatives helps bridge the gap between the concrete and abstract and strengthens the ability to generalize and transfer ideas. The process enables students to recognize and use patterns as a problem-solving tool and creates opportunities for them to explore, justify,
represent, solve, construct, investigate, and predict. This strategy allows for different levels of
cognitive development and encourages students to think about concepts while working with the
materials. Manipulatives promote communication of ideas and provide a problem-solving and decision-
making focus linked to real-world learning.

Media Presentation
A media presentation involves the use of various media to present information and ideas.
The presentation could involve such formats as audiotape or videotape, presentation slide shows, or
multimedia graphics using sound systems, television or computer monitors, overhead projections. The
presentations can increase interest in a topic by providing currency and variety and by appealing to
different learning styles, such as visual-spatial styles. Presentations may be interactive and involve
some form of audience participation. Presentation graphics software automates the creation of visual
aids for lectures, skills training sessions, and group presentations, often in the form of colourful and
animated slides and handouts.

Assessment Strategies/Recording Devices

Assessment Strategies:
Observation
Observation is a process of systematically viewing and recording student behaviour for the purpose of
making programming decisions. Observation can take place anytime or in any setting, to assist the
teacher in making the decisions required for effective teaching.

Assessment Recording Devices:
Anecdotal Record
The anecdotal record is a short narrative describing both a student behaviour and the context in which
the behaviour occurred. An anecdotal record should objectively report specific and observed
behaviours and may describe student performance in detail in writing. An anecdotal record provides
rich portraits of an individual student’s achievement and includes observations that have special
significance and cannot be obtained from other classroom assessment strategies.

Checklist
The checklist is a list of actions, descriptions, skills, concepts, behaviours, processes, and/or attitudes
that a rater (teacher) checks off as he or she observes them. A checklist includes a written list of
performance criteria that are used to assess student performance through observation, or to assess
written work or other product forms such as oral presentations, art and media works, and models. A
checklist is a diagnostic, reusable tool that can be helpful in charting student progress.

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<td>_ Staying on task _ _ Participating _ _ Working together</td>
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<tr>
<td>_ Listening to others _ _ Good Time Management _ _ Focused</td>
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<tr>
<td>_ Trying different methods _ _ Discovering Solutions</td>
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References
• http://www.curriculum.org/lms/files/tips4rm/gr9Unit1.pdf#search=%22MG2.03%22
• Ontario Curriculum Unit Planner 3.0 Runtime